On the bremsstrahlung cross section in ep and e-Au

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1 Introduction

A question has been raised on the amount of bremsstrahlung cross section in ep and e-Au operation and its implications to electron beam lifetime. This document gives an estimate on magnitude of the Bethe-Heitler cross section across all energies considered, and calculates mean number of bremsstrahlung photons per one bunch crossing.

The cross sections are evaluated for minimal bremsstrahlung photon energy given by 1% of electron beam energy, as such energy loss is still tolerated for the electron to remain in the beam.

2 Bethe-Heitler bremsstrahlung cross section

In ultra-relativistic approximation the cross section integrated over photon angles is given by Eq. 93.17 in QED texbook by Berestetskii, Lifshitz and Pitaevskii, 1982:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\omega} = 4Z^2\alpha r_e^2 \frac{1}{\omega}\frac{\varepsilon'}{\varepsilon}\left(\frac{\varepsilon}{\varepsilon'}+\frac{\varepsilon'}{\varepsilon}-\frac{2}{3}\right)\left(\ln\frac{2\varepsilon\varepsilon'}{m_e\omega}-\frac{1}{2}\right), \tag{1}$$

where ω is photon energy and ε and ε' is initial and final electron energy respectively. All ω, ε and ε' energies are given in target proton/nucleus rest frame. Z = 79 for gold and 1 for proton, m_e is electron rest mass and $\alpha r_e^2 = 0.57946$ mb.

3 Luminosity per one bunch crossing

The procedure on converting instantaneous luminosity L in $\mathrm{cm}^{-2}\mathrm{s}^{-1}$ to luminosity per one bunch crossing $\mathscr{L}_b$ in mb^{-1} was outlined by Bill in his talk about tagger multiplicity at far-forward detectors meeting on April 27, 2020 [1].

With n_b the number of bunches and using conversion $1\mathrm{mb} = 10^{-27}\mathrm{cm}^2$, it is

$$\mathscr{L}_b = 10^{-27} L \frac{1}{n_b}\frac{l}{\beta c} \tag{2}$$

where β is velocity of the beam, $l = 3834\,\mathrm{m}$ is the machine circumference and $c = 2.99792\times 10^8\ \mathrm{m\,s^{-1}}$ is the speed of light.

The $l/(\beta c)$ is period of one orbit, about 13 µs at the top ep energy. When divided by number of bunches it gives bunch spacing in time around the orbit. Values of β for electron and nuclear beams are compatible with each other.

4 Results

Using beam energies, number of bunches and values of luminosity from Tab. 3.3 and Tab. 3.5 for ep and eAu operation, as given in parameter tables [2], the total bremsstrahlung cross section σ_{BH} is found by integrating Eq. 1 from minimal photon energy $E_{\gamma,\mathrm{min}}$ up to electron beam energy, and luminosity per bunch crossing $\mathscr{L}_b$ is obtained by Eq. 2. Mean number of bremsstrahlung photons in one bunch crossing is then calculated as $\lambda_{\mathrm{phot}} = \sigma_{\mathrm{BH}} \times \mathscr{L}_b$.

$E_{\gamma,\mathrm{min}}$ is set as 1% of electron beam energy, as it is tolerated energy loss for an electron to remain in the beam.

The results are summarized in Tab. 1 for ep beams and in Tab. 2 for e-Au beams.

The first section in both tables is input from Tab. 3.3 and Tab. 3.5 in [2] for ep and e-Au beams respectively. They contain proton, Au ion and electron beam energies, number of bunches n_b and luminosity L.

Luminosity for e-Au in Tab. 3.5 in parameter tables has a meaning of luminosity per nucleon, L_{eN}. It is related to observed luminosity L as $L = L_{eN}/A$. The L is then input to Eq. 2 to get luminosity per bunch crossing and mean number of photons in e-Au Tab. 2.

Results are in the second part:

- γ_p, γ_{ion} and γ_{e} are Lorentz factors of the beams
- $E_{\gamma,\mathrm{min}}$ is minimal energy of bremsstrahlung photon
- σ_{BH} is the total Bethe-Heitler bremsstrahlung cross section for a given $E_{\gamma,\mathrm{min}}$
- $\mathscr{L}_b$ is luminosity per bunch crossing
- $\lambda_{\mathrm{phot}} = \sigma_{\mathrm{BH}} \times \mathscr{L}_b$ is mean number of bremsstrahlung photons in one bunch crossing

Species	p	electron	p	electron	p	electron	p	electron	p	electron
Energy [GeV]	275	18	275	10	100	10	100	5	41	5
n_b	290		1160		1160		1160		1160	
L [10^{33} cm^{-2}s^{-1}]	1.65		10.05		4.35		3.16		0.44	
γ_{p}	293.1		293.1		106.6		106.6		43.7	
γ_{e}	35225.1		19569.5		19569.5		9784.8		9784.8	
$E_{\gamma,\mathrm{min}}$ [GeV]	0.18		0.1		0.1		0.05		0.05	
σ_{BH} [mb]	236.8		229.6		217.1		208.5		197.5	
$\mathscr{L}_b$ [mb^{-1}]	0.073		0.111		0.048		0.035		0.005	
λ_{phot}	17.2		25.4		10.4		7.3		1.0	

Table 1 Bremsstrahlung cross section and mean number of photons in electron-proton collisions

Species	Au ion	electron	Au ion	electron	Au ion	electron	Au ion	electron
Energy [GeV]	110	18	110	10	110	5	41	5
n_b	290		1160		1160		1160	
L_{eN} [10^{33} cm^{-2}s^{-1}]	0.59		4.76		4.77		1.67	
L [10^{31} cm^{-2}s^{-1}]	0.3		2.42		2.42		0.85	
γ_{ion}	117.5		117.5		117.5		43.7	
γ_{e}	35225.1		19569.5		9784.8		9784.8	
$E_{\gamma,\mathrm{min}}$ [GeV]	0.18		0.1		0.05		0.05	
σ_{BH} [kb]	1.41		1.36		1.31		1.23	
$\mathscr{L}_b$ [b^{-1}]	0.132		0.266		0.267		0.093	
λ_{phot}	186		363		349		115	

Table 2 Bremsstrahlung cross section and mean number of photons in e-Au

5 Summary

Notably large cross sections have been found in the e-Au case, reaching units of kb. Corresponding mean numbers of bremsstrahlung photons reach hundreds of photons emitted by electrons lost from the beam in one bunch crossing.

Cross section formula in Eq. 1 follows from Born approximation. The exact theory introduces a correction which depends on αZ and reduces the logarithmic term.

The Lorentz factors of the beams were obtained only to compare the initial kinematics with the study done by Krzysztof Piotrzkowski. In that study the cross section for e-Au beams at 110×10 GeV ranges from 1.58 kb to 1.86 kb, depending on model parameters. It is approximately compatible with cross section of 1.36 kb found here.

Values of λ_{phot} still underestimate photon multiplicities incident on photon detector (and exit window) of the luminosity monitoring system. Geant4 simulations show that with 4 X_0 thick graphite filter in front of the detector, like ZEUS had, photons at 1 MeV start to give a nonzero energy deposition in the detector. Minimal photon energies used here are larger, as they are motivated by requirements on the electron beam.

References

[1] W. Schmidke, "*Occupancy in low-Q2 tagger*," *Forward Detector/IR Integration, April 27, 2020* . `https://indico.bnl.gov/event/8379/contributions/36983/attachments/27781/42618/FFdet_27.04.20.pdf`.

[2] "*EIC parameter tables*,". `https://www.dropbox.com/s/us4pubqihs4ctdr/EIC-parameter.tables.pdf?dl=0`.