Range of Beta Rays

There is an ancient bit of wisdom regarding the range of beta's called the Feather relation, which I always remembered as the range in $\mathrm{gm}/\mathrm{cm}^2$ is about half the energy in MeV. Actually, what Feather (1938) determined empirically was

$$\mathrm{R}\left(\mathrm{gm}/\mathrm{cm}^2\right) \approx 0.543\,E - 0.160 \text{ for } E > 0.8\,\mathrm{MeV}$$

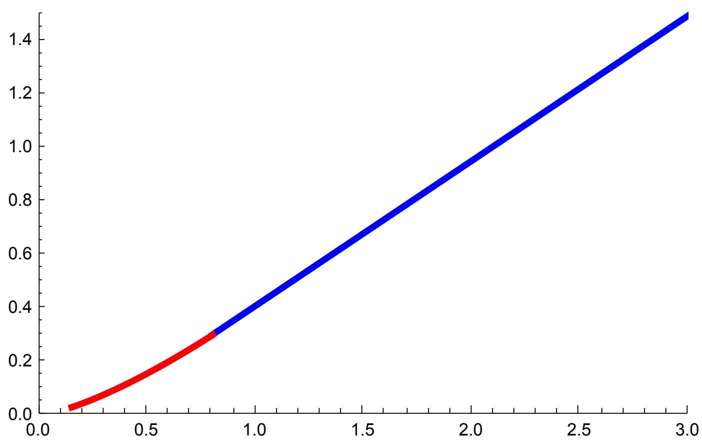
This was 'improved' by Glendenin and Coryell in 1946 to

$$R\left(\mathrm{gm}/\mathrm{cm}^2\right) \approx \begin{array}{ll} 0.542\,E - 0.133 & E > 0.8\,\mathrm{MeV}) \\ 0.407\,E^{1.38} & 0.15 < E < 0.8\,\mathrm{MeV} \end{array}$$

Both of these expressions appear in Fermi's *Nuclear Physics* notes (Chicago, 1949). The second form is also contained in the Segre's *Nuclei and Particles* (Addison-Wesley, 2nd ed., 1977), which has a strong overlap with much of Fermi's notes.

Here is a plot of the improved version:

```
eRange =
  Plot[ 0.407 Energy^1.38, {Energy, 0.15, 0.8}, PlotStyle -> {RGBColor[1, 0, 0], Thickness[0.01]}];
eFeather = Plot[0.542 Energy - 0.133 , {Energy, 0.8, 3},
   PlotStyle -> {RGBColor[0, 0, 1], Thickness[0.01]}, PlotRange → {{0.0, 3.0}, {0.0, 1.5}}];
Show[eFeather, eRange]
```



which shows that these empirical results at least have the nice property of being continuous...

For reasons too complicated to detail here, I found it necessary to integrate the dE/dx formula for energy loss to find the actual range. Below the minimum ionizing energy (for electrons, this is probably around 3 MeV), one has rather accurately

$$\frac{dE}{dx} \approx \frac{C}{\beta^2}$$

Where C is some constant of order $1.6\ \mathrm{MeV}/\left(\mathrm{gm}/\mathrm{cm}^2\right)$. Assuming this holds all the way down, we can then find the range by integrating:

$$dx = \frac{1}{C}\beta^2\, dE = \frac{1}{C}\frac{p^2}{E^2}\, dE = \frac{1}{C}\frac{E^2-m^2}{E^2}\, dE$$

$$\Rightarrow R = \frac{1}{C}\int_m^E\left(1-\frac{m^2}{E^2}\right) dE = \frac{1}{C}\left(E+\frac{m^2}{E}\right)\Big|_m^E$$
$$= \frac{1}{C}\left\{(E-m)+\left(\frac{m^2}{E}-m\right)\right\} = \frac{1}{C}\frac{(E-m)^2}{E}$$

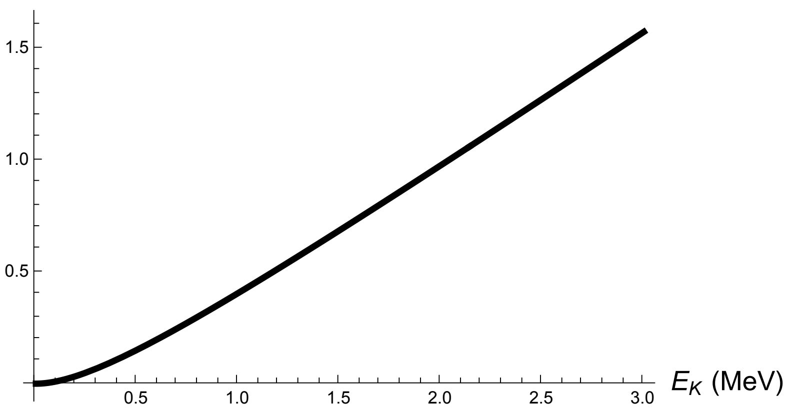
or in terms of electron kinetic energy (which is what is commonly quoted when dealing with low energy betas, and in fact is what appears in all of the above empirical range formulas:

$$R = \frac{1}{C}\frac{{E_K}^2}{E_K+m}$$

This is the the 'non-empirical' form I've 'derived'; how does it compare to the empirical results? Taking C to be the observed minimum for electron energy loss of about 1.63 MeV / $(\mathrm{gm}/\mathrm{cm}^2)$, I get

```
myRange = 1/1.63 Energy^2/(Energy + 0.511); myRangePlot = Plot[myRange, {Energy, 0, 3},
  PlotStyle → {RGBColor[0, 0, 0], Thickness[0.01]}, AxesLabel -> {"E_K (MeV)", "Range (gm/cm^2)"}]
```

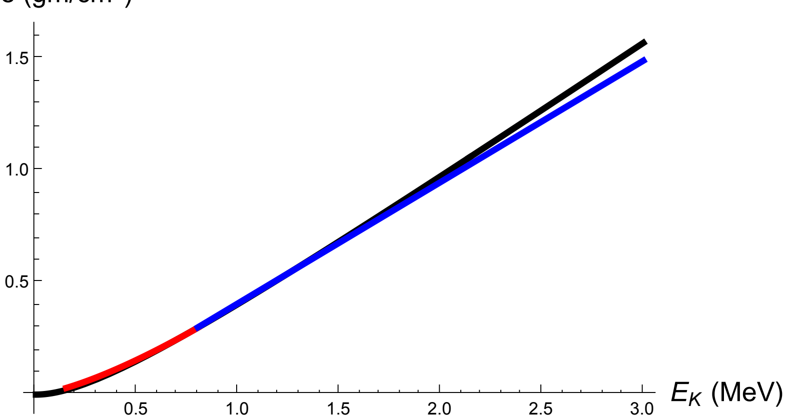
Range (gm/cm^2)



which compares embarrassingly well with the empirical curve of Feather as modified by Glendenin and Coryell:

```
Show[myRangePlot, eRange, eFeather]
```

Range (gm/cm^2)

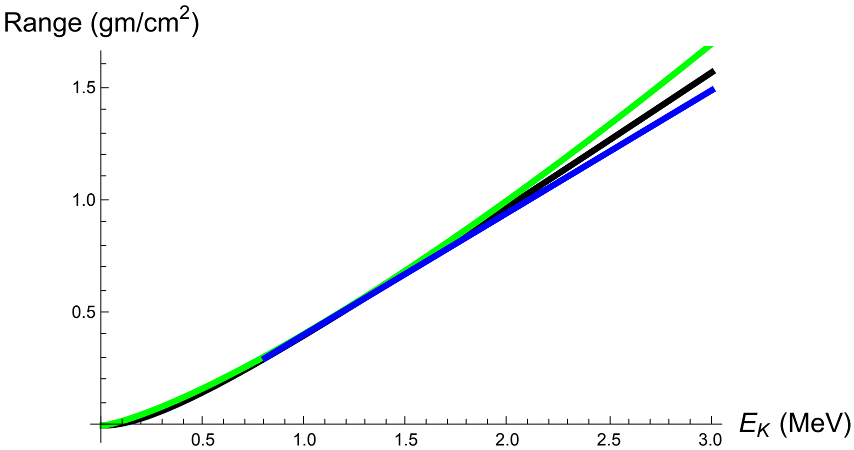


Thus emboldened, I dedide to compare my result (in black above) with the another, more 'modern' parameterization given in the manual for the undergraduate beta-gamma lab:

$R\left(\text{gm}/\text{cm}^2\right) = 0.412\,E^{1.29}$ (L. Katz and A.S. Penfold, Rev. Mod. Phys. 24, 1 (1952)).

In fact, let's compare this (in green), the blue+red "Feather++", and my result (in black):

```
eLab = Plot[ 0.412 Energy^1.29, {Energy, 0.0, 3.0},
  PlotStyle -> {RGBColor[0, 1, 0], Thickness[0.01]}]; Show[myRangePlot, eLab, eFeather]
```



At this stage, I have no reason to favor my approach over the result given in the lab manual, but Segre provides a range plot that goes up to 10 MeV (where the range is supposed to be 5 gm/cm^2in Al). Let's compare:

```
LabManualRange = 0.412 Energy^1.29 /. Energy -> 10
```

8.03336

$$\text{myFormulaRange} = \frac{1}{1.63}\,\frac{\text{Energy}^2}{\text{Energy}+0.511}\ /.\ \text{Energy} \to 10$$

5.83671

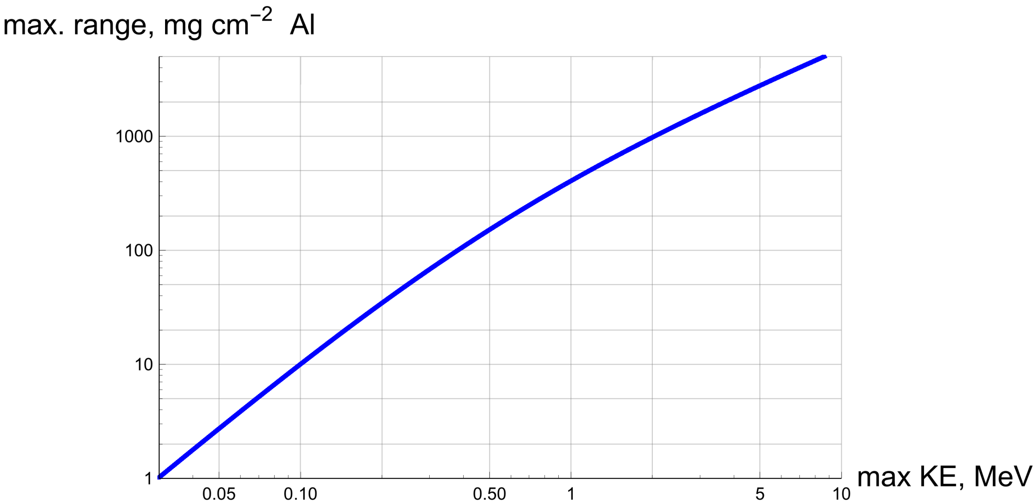
OK, I'm not perfect, but doing a lot better than the lab manual result. BTW, the Feather result, which is below mine at 3 MeV, does not fare well at large energies (nor should it; it's advertised as applying only below 3 MeV):

```
FeatherRange = 0.407 Energy^1.38 /. Energy -> 10
```

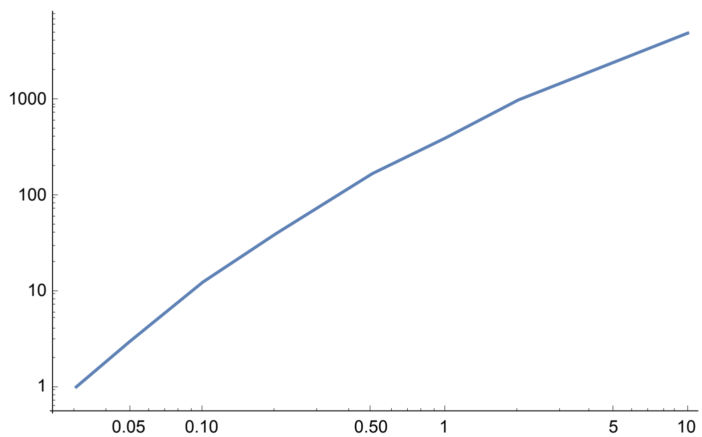
9.76325

Let's make that plot in Segre (Figure 2-13) using my result

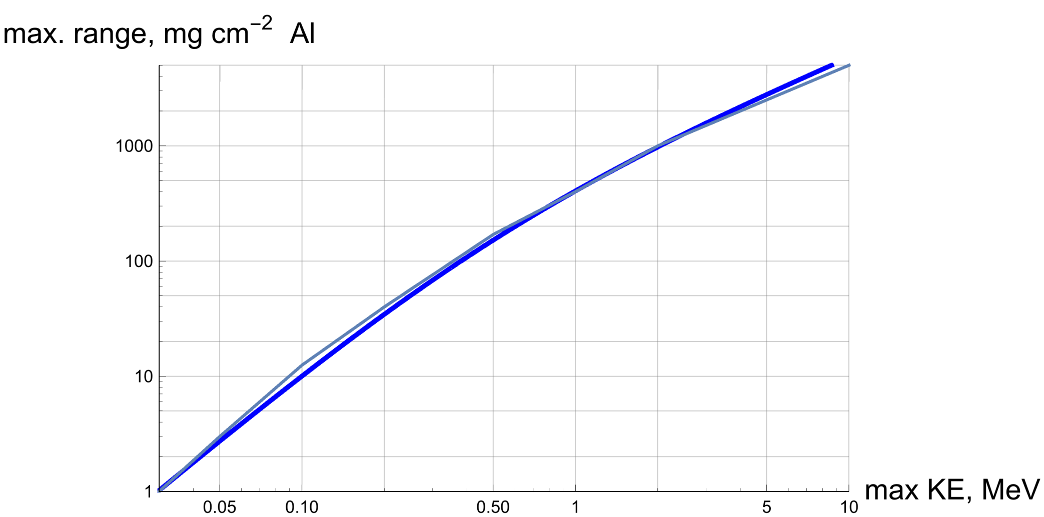
```
segrePlot = LogLogPlot[1000 myRange, {Energy, 0.01, 10},
  PlotRange -> {{0.03, 10}, {1, 5000}}, PlotPoints -> 100, GridLines ->
   {{0.05, 0.1, 0.2, 0.5, 1, 2, 5, 10}, {2, 5, 10, 20, 50, 100, 200, 500, 1000, 2000, 5000}},
  PlotStyle -> {Thickness[0.007], RGBColor[0, 0, 1]},
  AxesLabel -> {"max KE, MeV", "max. range, mg cm^-2  Al"}]
```




```
segreValues = {{0.03, 1}, {0.05, 3}, {0.1, 12.5},
   {0.2, 40}, {0.5, 170}, {1, 400}, {2, 1000}, {5, 2500}, {10, 5000}};
segreValuesPlot = ListLogLogPlot[segreValues, Joined → True]
```




```
Show[segrePlot, segreValuesPlot]
```



Gratifying agreement...

(The reference for this figure in Segre is wrong, I infer that it must be *Beta and Gamma Ray Spectroscopy*, Kai Siegbahn, North-Holland, Amsterdam, 2nd ed. 1965.)

Practical Application:

(Yes, there is one.)
Suppose you want to test the efficiency of a scintillator paddle. The standard way to do so is to place it between two other paddles. and compare the triple coincidence $P_{\text{TOP}} \cdot P_{\text{MIDDLE}} \cdot P_{\text{BOTTOM}}$ to the flux through the stack given by $P_{\text{TOP}} \cdot P_{\text{BOTTOM}}$. You can do this with cosmics if you are very patient, but it's faster to use a source. Just about the only beta source with a good change of getting through three paddles is Ruthenium-106, with an endpoint of 3.5 MeV. The above figure tells you the range for 3.5 MeV beta's is about 2 gm / cm^2, which is about 2 cm of scintillator. So it works-on paper. In the real world, the counting rate goes to zero at the endpoint, so you’ll definitely want a thinner stack than 2 cm.